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Measurement of orbital angular momentum with an off-axis superposition of vector modes

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Abstract
We propose an off-axis superposition of vector modes with orthogonal polarizations, constructed from a general scalar helical vortex mode with unknown topological charge m, as a method to measure its orbital angular momentum. We derived analytic expressions for sets of solutions to find lines of linear polarization (L lines) within the composite polarization field. We found that the solutions corresponding to the angular component of the composite field depend only on the displacement of the beams and the topological charge m, and they are invariant under propagation and changes in the relative amplitude and phase between the beams.

Keywords: optical vortices, angular momentum, Mach–Zehnder interferometer, vector modes

(Some figures may appear in colour only in the online journal)

One of the most interesting and exploited properties of light is that it can carry orbital angular momentum (OAM) and transfer it to matter. A light beam with an azimuthal phase dependence $\exp(\mathrm{i}m\theta)$ presents a phase singularity known as *optical vortex* with a topological charge m and carries an OAM of $m\hbar$ per photon [1–4]. The OAM of light is receiving increasing attention, not only as an intrinsic property of light, but also as an experimental resource both in classical and quantum optics with many different applications, such as optical tweezers and micromanipulation [5–8], optical communications [9], quantum information science [10–13], and optical vortices [14–18], among others.

Circular and linear polarization singularities are the vector counterpart of phase singularities in scalar fields [19–22] and are a natural occurrence in spatially inhomogeneous polarization patterns such as Full Poincaré beams [23, 24]. Although creating beams containing optical vortices or carrying OAM is relatively straightforward, measuring the topological charge m of these vortices is not always so, and different methods are used to do it, including interference [25, 26], diffraction [27–35], mode transformations [36–39], modal decomposition [40–42], and interferometers [43–46].

We propose a new interferometer-based method to measure the OAM of a general scalar helical vortex mode of unknown topological charge m by using a superposition of auxiliary displaced vector modes with orthogonal polarizations generated from the scalar vortex beam, and finding the lines of linear polarization (L lines) within the composite polarization field with the Stokes parameters. Furthermore, we built an experimental setup to verify that for a doughnut Laguerre–Gaussian beam of topological charge m we can optically construct the corresponding vector modes and use the aforementioned superposition to measure the OAM content of the beam.

Consider an helical vortex mode $\psi_m(\boldsymbol{r}, z)$ of the general form

$$\psi_m(\boldsymbol{r}, z) = f(r, z)\exp(\mathrm{i}m\theta), \tag{1}$$

where $r=\sqrt{x^2+y^2}$ and $\theta=\tan^{-1}(y/x)$ are the transverse polar coordinates, x and y are given in units of beam waists w_0, m is the unknown topological charge and $f(r,z)$ is the radial profile of the helical vortex mode. We assume $\psi_m(\boldsymbol{r},z)$ is square integrable in the transverse plane, $f(r,z)$ is any complex function with $f(0,z)=0$, and it fully describes the propagation of the field.

In order to determine the topological charge m of the beam in (1), we first construct two vector modes with orthogonal polarizations from the scalar field $\psi_m(\boldsymbol{r},z)$, which in the circular polarization basis $\{\hat{\boldsymbol{c}}_+,\hat{\boldsymbol{c}}_-\}$ are given by

$$\boldsymbol{U}^m_{\mathrm{VM}}(\boldsymbol{r},z)=\psi_{-m}(\boldsymbol{r},z)\hat{\boldsymbol{c}}_+ + \psi_m(\boldsymbol{r},z)\hat{\boldsymbol{c}}_-, \tag{2}$$

$$\boldsymbol{U}^m_{\mathrm{VM}\perp}(\boldsymbol{r},z)=-\mathrm{i}\,\psi_{-m}(\boldsymbol{r},z)\hat{\boldsymbol{c}}_+ + \mathrm{i}\,\psi_m(\boldsymbol{r},z)\hat{\boldsymbol{c}}_-, \tag{3}$$

where $\hat{\boldsymbol{c}}_\pm=(\hat{\boldsymbol{x}}\pm\mathrm{i}\,\hat{\boldsymbol{y}})/\sqrt{2}$ are the unit vectors of the circular polarization basis. For $m=1$, the auxiliary vector modes $\boldsymbol{U}^m_{\mathrm{VM}}$ and $\boldsymbol{U}^m_{\mathrm{VM}\perp}$ reduce to cylindrical vector beams with radial and azimuthal polarizations, respectively. We now define a composite polarization field obtained from the off-axis superposition of $\boldsymbol{U}^m_{\mathrm{VM}}(\boldsymbol{r},z)$ and $\boldsymbol{U}^m_{\mathrm{VM}\perp}(\boldsymbol{r},z)$,

$$\boldsymbol{E}(\boldsymbol{r},z)=\mathrm{e}^{\mathrm{i}\beta}\,\boldsymbol{U}^m_{\mathrm{VM}}(\boldsymbol{r}-\boldsymbol{r}_0,z)+\xi\,\boldsymbol{U}^m_{\mathrm{VM}\perp}(\boldsymbol{r}+\boldsymbol{r}_0,z), \tag{4}$$

where $0\le\beta<2\pi$ is the phase difference between the beams, $0<\xi\le 1$ is their relative amplitude, and $\boldsymbol{r}_0=(x_0,y_0)$ is the transverse displacement vector. The superposition of displaced vector modes with orthogonal polarizations generates a complex polarization structure which depends on the relative amplitude ξ, the phase difference β, and the total displacement distance $r_0=\sqrt{x_0^2+y_0^2}$ of each beam.

We use the Stokes parameters to study the polarization structure of the composite field $\boldsymbol{E}(\boldsymbol{r},z)$, which in the circular basis are

$$\begin{aligned} S_0&=|E_+|^2+|E_-|^2, & S_1&=2\mathrm{Re}\{E_+^*E_-\},\\ S_2&=2\mathrm{Im}\{E_+^*E_-\}, & S_3&=|E_+|^2-|E_-|^2, \end{aligned} \tag{5}$$

where E_+ and E_- are the circular components of $\boldsymbol{E}(\boldsymbol{r},z)$ which are obtained from (2) to (4). The normalized Stokes parameters $[S_1/S_0,\ S_2/S_0,\ S_3/S_0]$ describe a point on the Poincaré sphere, where the North (South) pole represents positive (negative) circular polarization and the equator is populated by linear polarization states.

To find the unknown topological charge m of the original scalar field $\psi_m(\boldsymbol{r},z)$, we consider the lines of linear polarization on planes transverse to the propagation axis, or L lines. Just as C-points are found by solving the set of equations $\{S_1=0,\ S_2=0\}$, L lines are given by the solutions to $S_3=0$. We found that for a beam of the form given in (1) the solutions to $S_3=0$ associated to $f(r,z)$ are given by

$$\mathrm{Im}\left\{\mathrm{e}^{\mathrm{i}\beta}f(|\boldsymbol{r}-\boldsymbol{r}_0|,z)\,f^*(|\boldsymbol{r}+\boldsymbol{r}_0|,z)\right\}=0, \tag{6}$$

whereas the solutions to $S_3=0$ associated to the angular component of the field can be written as

$$(x-x_0\tan\Omega_N)^2+(y+y_0\tan\Omega_N)^2=r_0^2\sec^2\Omega_N, \tag{7}$$

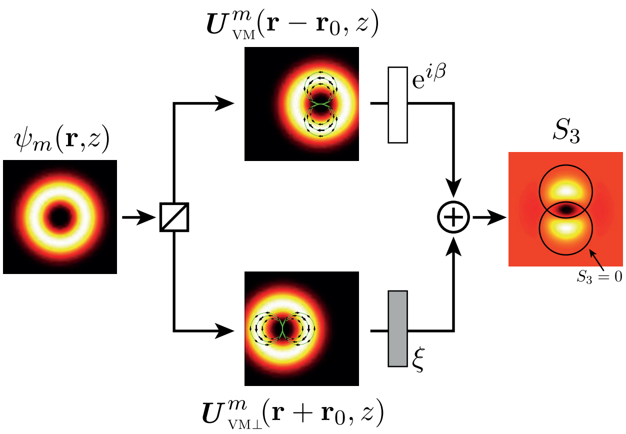

Figure 1. Schematic representation of the off-axis superposition of orthogonal doughnut vector modes to find the topological charge m.

where $\Omega_N=\pi N/2m$ and $N=m-1, m-3,\ldots,-(m-3), -(m-1)$.

We can see from (7) that the solutions to $S_3=0$ associated to the angular component of the field describe circles. The circles have different radii and centres, which are independent of the phase difference β and the relative amplitude ξ, and are a function only of the displacement of the beams and the topological charge m. All the circles intersect at the position of the beam centroids $\pm(x_0,y_0)$.

The results presented so far are valid for any general helical vortex mode which can be written in the form given in (1). For simplicity we now focus on doughnut or single-ringed vortex beams, so that $f(r,z)$ can be any complex function which has a single radial maximum for some $r>0$. We can see in (6) that, for any allowed complex function $f(r,z)$ such that $f(0,z)=0$ is the only zero in the transverse radial coordinate, its solutions also intersect at the centroids of the beams, and since the propagation behaviour is completely described by $f(r,z)$, the circles in (7) are invariant under propagation. Figure 1 shows a flow chart of how we can use these results to find the topological charge m of the scalar field $\psi_m(\boldsymbol{r},z)$.

Figure 2 shows the angular solutions to $S_3=0$ given in (7) for a superposition of orthogonal vector modes of angular orders $m=1$ through $m=4$ with a horizontal displacement of $x_0=1/2$. We can see that for each value of m we have exactly m circles, all of which intersect at the positions of the centre of the beams, so to know the angular order m of the beams, we need only count the number of circles in the $S_3=0$ curves. Even though the relative amplitude term ξ is not present in (6) and (7), we introduced it into our model to allow us to consider the additional degree of freedom and better describe the experimental implementation. Since the centres and radii of the circles are independent of both ξ and β, experimentally we would be able to observe the circles regardless of any unbalanced intensity or relative phase (difference in optical path) between the interfering beams.

Now we consider the scalar doughnut beam to be a Laguerre–Gaussian (LG) beam of radial order $p=0$ and angular order m. Since (7) shows that the circles

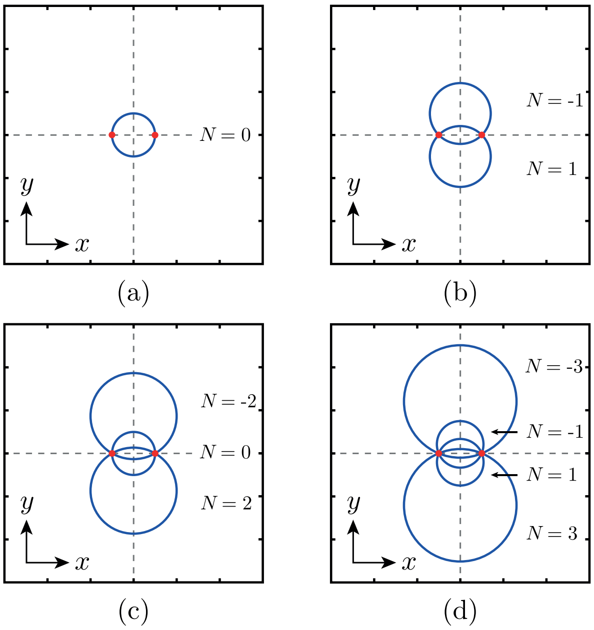

Figure 2. Solutions of $S_3 = 0$ for a superposition of orthogonal vector modes with a horizontal displacement of $x_0 = 1/2$ for topological charges (a) $m = 1$, (b) $m = 2$, (c) $m = 3$, and (d) $m = 4$.

associated with the angular component of the field are propagation-invariant, without any loss of generality we consider the LG field at $z = 0$, such that the radial profile is given by

$$f(r, 0) = r^{|m|} \exp\left(-r^2\right). \tag{8}$$

We write the radial distribution (8) in Cartesian coordinates, substitute into (6), and after carrying out some algebra we get

$$[(x - x_0)^2 + (y - y_0)^2][(x + x_0)^2 + (y + y_0)^2] = 0, \tag{9}$$

which for real displacements only allows the solutions $(x, y) = \pm(x_0, y_0)$, i.e. the positions of the displaced beam centroids.

Figure 3 shows our experimental setup to find the topological charge of doughnut vortex beams. To generate the doughnut LG beams, we shine a diagonally-polarized fundamental Gaussian beam onto a spiral phase plate (SPP) of charge m and use a linear polarizer (LP1) to ensure we have a 45° polarization. Next we split the beam with a polarizing beam splitter (PBS) and one of the resulting beams passes through a Dove prism which transforms its angular momentum as $m\hbar \rightarrow -m\hbar$ [47], and then we use two quarter-wave plates (QWP) to ensure we have a positive circular polarization in the interferometer arm containing the Dove prism and a negative circular polarization in the remaining arm. We recombine both arms of the interferometer using a beam splitter (BS1) to obtain the LG vector mode (VM) from (2) at point (1) in figure 3. The vector mode from (1) is then fed into a second interferometer which generates the orthogonal vector mode (VM⊥) in one of its arms by placing two half-wave plates (HWP) with their fast-axes at an angle of 45° between them. A neutral filter can be introduced into one of the interferometer arms to change the relative amplitude between the orthogonal components of the superposition. The displacement and the phase difference between the beams are introduced by moving one of the beam

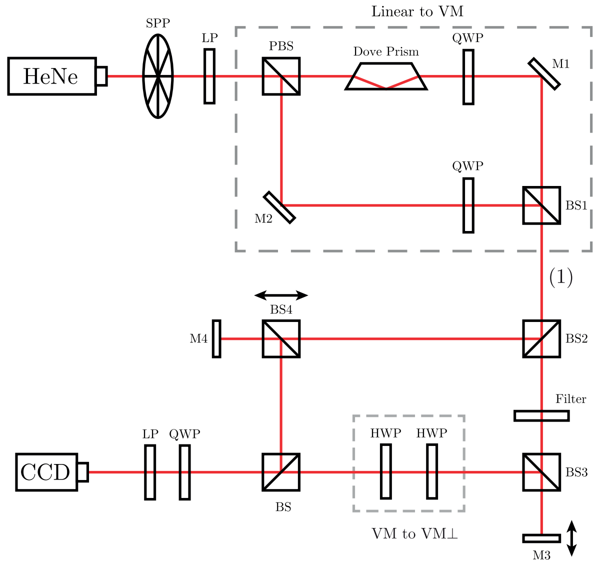

Figure 3. Experimental setup to generate the off-axis superposition of orthogonal vector modes.

splitters (BS4) and one of the balancing mirrors (M3). The final polarizer and quarter-wave plate are used to obtain the Stokes parameters of the composite beam. Even though our method is insensitive to variations in the relative intensity and phase between the auxiliary vector modes, the experimental setup is highly sensitive to misalignments and tilting of optical elements, which can prove difficult to detect at first. The high frequency noise in the experimental Stokes parameters was reduced using a low-pass Gaussian filter.

Figure 4(a) shows the theoretical Stokes parameters S_0 and S_3 of a balanced superposition of LG vector modes with topological charge $m = 1$ and $\beta = \pi$, whereas figure 4(b) shows their experimental counterparts. We can see from figure 4 that for a topological charge m, we can experimentally observe the corresponding m circles in the Stokes parameter S_3 and thus find out the value of the orbital angular momentum of the beam.

In conclusion, we found that for a scalar doughnut beam containing an optical vortex of unknown topological charge m, we can use a superposition of displaced orthogonal vector modes generated from the aforementioned doughnut beam to observe the corresponding Stokes parameter S_3, and by finding the curves for $S_3 = 0$ we can obtain the topological charge m of the original beam. Furthermore, we proved that the curves for $S_3 = 0$ remain invariant through propagation and changes in relative phase and amplitude between the orthogonal components of the composite polarization field for any doughnut beam, as long as its angular component can be written as a separate factor from its radial and z components. Finally, we proposed an experimental setup to generate such a superposition and we measured the topological charge of a beam generated by a spiral phase plate. We compared the theoretical and experimental results for a LG beam of angular order $m = 1$, and we observed that the corresponding $S_3 = 0$ circle appeared as expected, although it was horizontally elongated due to aberration effects from the system. For instance, for angular order $m = 2$ we would expect to see

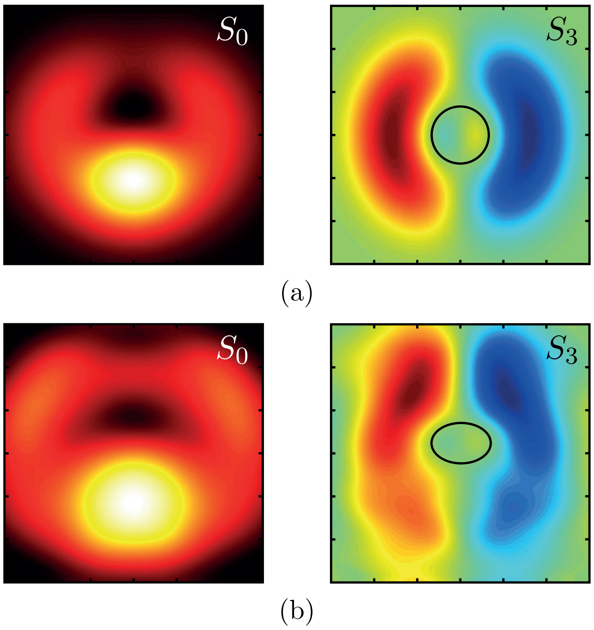

Figure 4. (a) Theoretical and (b) experimental Stokes parameters S_0 and S_3 for a balanced off-axis superposition of orthogonal LG vector modes with topological charge $m = 1$ and phase difference $\beta = \pi$.

the two corresponding circles as in figure 2(b) with the same horizontal elongation due to aberration effects.

Our results shown in (6) and (7) hold for any helical vortex mode, including multi-ringed beams such as Bessel–Gauss beams and LG beams of radial order $p > 0$, and for ideal nondiffracting Bessel beams even if they are not square integrable. For $\psi(\boldsymbol{r}, z)$ a multi-ringed vortex beam, $f(r, z)$ has additional zeros at some values of $r > 0$ in the transverse radial coordinate which show up as two sets of concentric circles, each one of them centred around one the centroids. Although the rings generated by $f(r, z)$ are also independent of ξ and β, only the circles associated with the angular part of the beam are propagation-invariant, and since the radial rings would make it virtually impossible to distinguish the angular circles, we conclude that our method is better suited for single-ringed vortex beams.

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