Engineering of nondiffracting beams with genetic algorithms

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We introduce a technique to generate arbitrary nondiffracting beams. Using a genetic algorithm that uses a Gaussian weight function merged with spatial spectrum engineering techniques, we show that it is possible to obtain the angular spectrum representation of arbitrary light patterns, thus demonstrating their nondiffracting properties. © 2012 Optical Society of America

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Beams whose transverse profile remain invariant during propagation, or nondiffracting beams (NBs), are useful in diverse areas [1], such as manipulation of biological and colloidal material [2] and soliton routing and steering [3]. Using group theory, it has been shown that there are only four fundamental families of NBs: plane waves, Bessel [4], Mathieu [5], and parabolic beams [6]. However, it is possible to generate NBs with arbitrary transverse profiles. The complex light field $\Psi(\mathbf{r})\exp(ik_z z)$ of a NB propagating along the z axis, without acceleration in the transverse plane, can be written using the Whittaker integral [4]

$$\Psi(\mathbf{r}) = \int_0^{2\pi} A(\phi)\exp[ik_t r\cos(\phi-\theta)]\mathrm{d}\phi. \quad (1)$$

Here k_z and k_t are the longitudinal and transverse components of wavenumber $k=(k_z^2+k_t^2)^{1/2}$, respectively, r and θ are the radial and azimuthal spatial transverse coordinates, respectively, ϕ is the azimuthal angle in the frequency domain, and $A(\phi)$ is the angular spectrum function that is defined along a circular-delta of radius k_t in the frequency space. The main problem for shaping NBs can be reduced to find the correspondence between the objective intensity function $|\Psi(\mathbf{r})|^2$ generated by $A(\phi)$, resulting in either patterns that have closed analytical expressions or in light patterns whose transverse profile can be integrated only by numerical methods like in the case of random [7] and fractional NBs [8]. In recent works [9,10], a technique based in a phase retrieval algorithm [11] has been used to generate arbitrary quasi-NBs.

In this Letter, we propose a method based on a genetic algorithm that allows one to obtain pure NBs by generating directly their corresponding complex angular spectrum functions, using the following angular spectrum function:

$$A(\phi) = \sum_{n=1}^{N}\alpha_n \exp(i\beta_n)\delta(\phi-\phi_n), \quad (2)$$

where α_n, β_n are the amplitude and phase parameters, respectively, of the angular spectrum or plane wave decomposition, δ is the Dirac delta function and $\phi_n = 2\pi n/N$. Substituting Eq. (2) into Eq. (1), we obtain

$$\Psi(\mathbf{r}) = \sum_{n=1}^{N}\alpha_n \exp(i\beta_n)\exp[ik_t r\cos(\phi_n-\theta)]. \quad (3)$$

Therefore the problem is reduced to find N complex coefficients that generate a light pattern whose intensity profile best fits a objective intensity profile $|\Psi_{\mathrm{obj}}(\mathbf{r})|^2$. The problem is not trivial because in general a modification in a particular α_n or β_n parameter produces changes in the complete spatial domain. Furthermore, some desired intensity patterns might not have a mathematically corresponding $A(\phi)$ function to generate them. Because of the complexity of this problem, we propose to use a genetic algorithm (GA) to solve Eq. (3). A GA is an optimization technique inspired on the evolutionary biological process of natural selection, which is capable of finding a global extremum of a given function to be optimized [12]. In optics, GAs have been used in diverse fields, such as tomographic image reconstruction [13], phase unwrapping [14], and photonic crystal slabs [15].

In order to initialize the GA, we choose a determined number of initial proposed solutions $\vec{\mu}$ or individuals, where each $\vec{\mu}$ represents a different angular spectrum. These individuals are formed by the amplitude and phase of the N coefficients $\vec{\mu} = \langle\alpha_1,\beta_1,\alpha_2,\beta_2,\ldots,\alpha_N,\beta_N\rangle$ where the parameters α_n and β_n are grouped together since they form a single complex coefficient. To compare the normalized objective $|\Psi_{\mathrm{obj}}|^2$ and the pattern Ψ obtained with the GA, we choose an error function of the form

$$\gamma = \sum_{j}^{N_x}\sum_{k}^{N_y} w_{jk}|\Psi\Psi^* - \Psi_{\mathrm{obj}}\Psi^*_{\mathrm{obj}}|, \quad (4)$$

where N_x and N_y are the number of points used to discretize the transverse plane. We propose the term w_{jk} as a weight function, being this is a step to ensure the convergence of the GA. We use a normalized Gaussian weighting function

$$w(\mathbf{r}) = \sigma_r^{-1}\sqrt{2/\pi}\exp(-r^2/2\sigma_r^2), \quad (5)$$

where σ_r is a parameter that is adjusted depending on the desired width of the beam. From a physical point of view, this Gaussian weight function is chosen because when a NB is apodized by a Gaussian transmittance, its respective ring of infinitesimal width in the angular spectrum representation becomes wider in radial direction but remains unaltered in azimuthal direction as it was demonstrated in [16]; thus the Gaussian transmittance produces a finite and scaled version of the pure NB, optimizing the convergence in the GA. Next, standard computational steps of GAs [12] are applied: We use a roulette wheel selection method, which is basically a random selection of individuals with probability of being chosen that is in inverse proportion to its γ value. As a next step, the chosen individuals are recombined by pairs using a one-point crossover scheme [12], producing two new individuals with their original data swapped for each pair combined. Then mutation is used to avoid stagnation: The GA selects a random component, either amplitude or phase, of a certain individual and replaces it by a random value taken from a nonuniform distribution. Finally, the GA uses elitism that refers to the replacement of the individuals that poses maxima γ values of the current generation with individuals that poses minima γ values of the former generation. The algorithm is iterated until we reach the condition of $\gamma < \varepsilon$, where ε is the desired tolerance. The iterations or generations p, required to reach a condition of $\log(\gamma) < -3$, is around 1000. The computation time required by the GA is in direct proportion to N value. In general, a low N value produces high γ values, while very high N values might slow down the convergence of GA.

As a first test of the GA before described, we aim to reconstruct the objective pattern shown in Fig. 1(a) whose transverse intensity profile is given by the superposition of two Bessel beams $|J_3(k_t r)[7\exp(i3\theta) + \exp(-i3\theta)]|^2$, being J_3 the third-order Bessel function. We use 40 plane waves in a transverse window of 100×100. The intensity and phase patterns obtained are shown in Figs. 1(b) and 1(c), respectively. Because of the random nature of the GA, an average of the error values according to the different iterations used by the GA is shown in Fig. 1(d). Contrary to the method developed in [9], the GA proposed here can recover the objective function starting even from an initial constant phase function. The angular spectrum of the intensity and phase functions are shown in Figs. 1(e) and 1(f), demonstrating that the pattern obtained is indeed a NB.

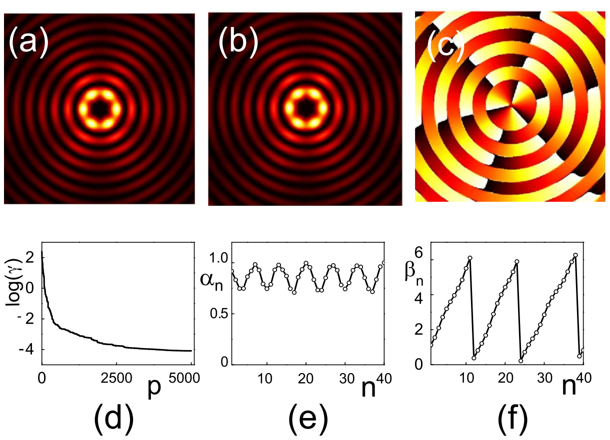

Fig. 1. (Color online) (a) Objective intensity pattern of a third order modulated Bessel beam and (b) intensity pattern obtained, using the genetic algorithm and (c) its phase distribution. All profiles are shown in a xy box of 12×12 with $k_t = 4$. (d) Evolution of the error function. (e) Amplitude and (f) phase angular spectrum obtained with the genetic algorithm.

With this procedure we show that it is possible to generate NBs that belong to the other fundamental families of NBs. We show in Figs. 2(b) and 2(c) and Figs. 2(h) and 2(i) the intensity and phase patterns obtained trying to recover a Mathieu [Fig. 2(a)] and parabolic [Fig. 2(g)] intensity profile, respectively, and we show their amplitude and phase angular spectrum representation in Figs. (2d)–(2e) and Figs. (2j)–(2k), respectively. Observe that for the objective Mathieu pattern, which corresponds to a purely real function, the GA also produces a good convergence as it is shown in the phase distribution recovered [Fig. 2(c)]. For the case of the parabolic beam, it is possible to accelerate the convergence process of the GA, by adjusting that Eq. (2) has all the initial intensity values equal to zero in one half of the angular spectrum ring. Hence if we have an insight of the possible angular spectrum representation, we can achieve a faster convergence. Therefore, as a final part of a general technique to produce complex NBs, a technique based on a phase retrieval technique [9] can be used to obtain first quasi-NBs, and then these patterns can be used as initial proposed solutions with the GA to finally obtain pure NBs.

By far the most exciting feature of our GA is the ability to obtain arbitrary NBs. Contrary to the quasi-NBs before obtained in [9], the number of useful intensity patterns

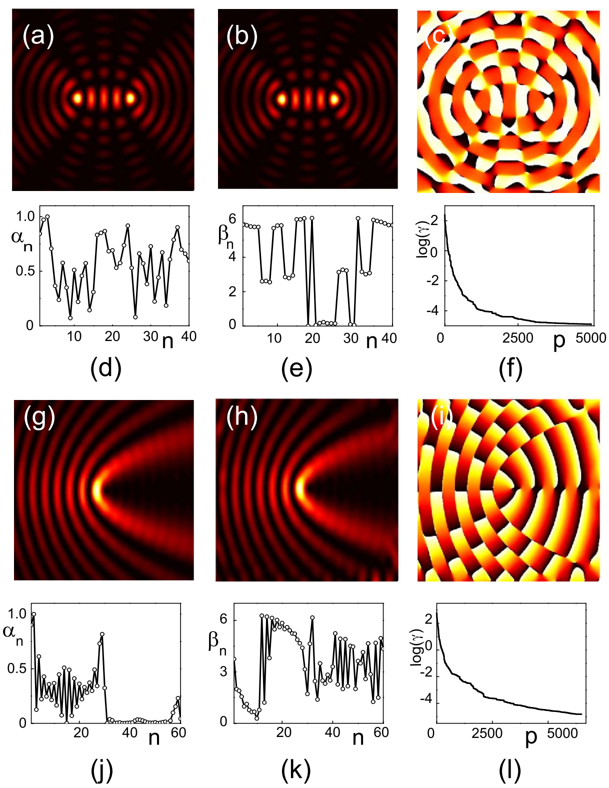

Fig. 2. (Color online) (a) Objective intensity pattern of a fourth even order Mathieu beam. (b) Intensity obtained pattern and (c) phase distribution using the GA. (d) Amplitude and (e) phase angular spectrum obtained using the GA. (f) Evolution of the error function. (g) Objective intensity pattern of a traveling parabolic beam. (h) Obtained intensity and (i) phase distributions using the GA. (j) Amplitude and (k) phase angular spectrum obtained using the GA. (l) Evolution of the error function. All profiles are shown in a xy box of 12×12 with $k_t = 4$.

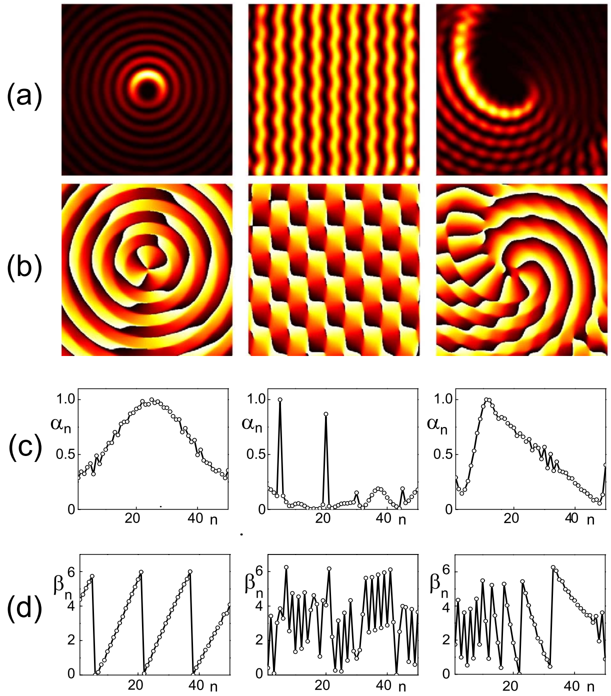

Fig. 3. (Color online) (a) Intensity, (b) phase, and (c) amplitude and (d) phase angular spectrum of unusual NBs that corresponds to an asymmetric Bessel beam, a zigzag cosine like beam and a spiral nondiffracting beam. All profiles are shown in a xy box of 12×12 with $k_t = 3$ for the zigzag case, and $k_t = 4$ for the rest.

are reduced, but these few intensity patterns are truly NBs, and hence their invariant condition is purely limited by the apodization in the experimental setup. The intensity and phase distributions of diverse complex NBs are shown in Figs. 3(a) and 3(b), respectively: an asymmetric Bessel beam that poses smooth continuity along their rings, a cosine like beam that bents periodically in a zigzag pattern, and a spiral NB. Observe that a similar zigzag pattern was introduced previously in [9] and [10] as a quasi-NB, but in this Letter we demonstrate that there is an ideal nondiffracting version. In Figs. 3(c) and 3(d) it is shown their correspondent amplitude and phase angular spectrum of these complex beams.

The apparent lack of periodicity in the angular spectrum for $\phi = 0 = 2\pi$ for certain cases in Figs. 3(c) and 3(d) is just because there can be an abrupt change between $n = 1$ and $n = N$ for α and β parameters; similar behavior was observed before for certain fractional NBs [8]. The analysis of all properties of the NBs here obtained are beyond the scope of this paper and will be introduced somewhere else.

In summary, we put forward a technique based on merging a GA that uses a weight Gaussian function with a phase retrieval based algorithm. This procedure allows the creation of complex transverse light patterns that are truly NBs. We demonstrate that all patterns here obtained are nondiffracting by showing their respective angular spectrum representation, and therefore their experimental observation is quite straightforward. This technique may find important applications where reconfigurable, complex optical lattices are important, such as the transport of bioparticles, manipulation of single atoms, and soliton routing and steering.

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