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Physica B 394 (2007) 351–356
PHYSICA B
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Spiraling solitons and multipole localized modes in nonlocal nonlinear media

Daniel Buccoliero[a,b], Servando Lopez-Aguayo[a,c], Stefan Skupin[b,d], Anton S. Desyatnikov[a], Ole Bang[e], Wieslaw Krolikowski[b,*], Yuri S. Kivshar[a]

[a]Nonlinear Physics Center, Research School of Physical Sciences and Engineering, Australian National University, Canberra, ACT 0200, Australia
[b]Laser Physics Center, Research School of Physical Sciences and Engineering, Australian National University, Canberra, ACT 0200, Australia
[c]Photonics and Mathematical Optics Group, Tecnologico de Monterrey, Monterrey 64849, México
[d]CEA-DAM/Ile de France, B.P. 12, 91680 Bruyeres-le-Chatel, France
[e]COM•DTU Department of Communications, Optics and Materials, Technical University of Denmark, DK-2800 Kongens Lyngby, Denmark

Abstract

We analyze the propagation of rotating multi-soliton localized structures in optical media with spatially nonlocal nonlinearity. We demonstrate that nonlocality stabilizes the azimuthal breakup of rotating dipole as well as multipole localized soliton modes. We compare the results for two different models of nonlocal nonlinearity and suggest that the stabilization mechanism is a generic property of a spatial nonlocal nonlinear response independent of its particular functional form.

PACS: 42.65.Tg; 42.65.Jx; 42.65.Sf

Keywords: Spatial solitons; Nonlocality; Nonlinear optics

1. Introduction

It is well known that nonlinear optical media support the formation of spatially localized structures—spatial optical solitons [1]. Nonlinearity of optical media is usually approximated by a local function of the light intensity assuming the refractive index change at a given spatial location depends solely on the light intensity at the same location. However, this local nonlinearity often supports only a certain type of stable optical solitons, such as the fundamental single-peak solitons. While in principle more complex optical structures such as multihump and vortex solitons can be predicted to exist as stationary states, such higher-order localized modes do not survive even weak perturbation and they break up into low-order solitons. For example, in the case of vortex solitons the ring-shaped beam with a helical phase structure disintegrates during the propagation in local nonlinear media due to the azimuthal instability [2].

However, as has been shown recently, the soliton stability may be enhanced dramatically if a nonlinear response of the medium is spatially nonlocal [3,4]. Nonlocality means that the change of the refractive index in a particular point is determined by the light intensity not only in the same point but also in its vicinity. Nonlocality appears naturally in many nonlinear systems including, for instance, self-action of laser beams in thermal media [5] and atomic vapors [6], as well as the dipole–dipole interaction of cold atoms in condensates [7].

Recent interest in the study of nonlocal nonlinear media has been stimulated by the experiments with spatial solitons in nematic liquid crystals [8] as well as the analysis of nonlocal interaction in dipolar Bose–Einstein condensates [7]. Various aspects of the impact of nonlocality on the propagation of finite-size beams and the formation of stable solitons in nonlocal media have been studied [9–12]. It has been shown that the stabilization of finite-size optical structures in nonlocal media is caused by the spatial averaging-out of low-scale intensity modulations, which results in the formation of broad and smooth waveguide

*Corresponding author.
E-mail address: wzk111@rsphysse.anu.edu.au (W. Krolikowski).

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doi:10.1016/j.physb.2006.12.063

structures guiding the beam and arresting its transverse instability. The stabilization of vortex solitons against symmetry-breaking instability was shown theoretically for two different models of nonlocal nonlinearity [3,4], as well as confirmed in experiment with thermal media [13].

It is therefore expected that nonlocality may stabilize other types of higher-order soliton structures, such as optical necklaces [14–16], soliton clusters [18], or recently introduced azimuthons [19]. The simplest azimuthon found to be highly unstable in local media [19] represents a single-charged vortex with a strong azimuthal intensity modulation such that it has a form of two localized peaks in its ring-shaped intensity profile. These peaks exhibit mutual angular rotation when the beam propagates. A similar object can be created as the coherent superposition of two closely placed fundamental (bell-shaped) solitons, with skewed initial trajectories. However, in local nonlinear media such bound state of two solitons is *always unstable*.

In this paper, we study rotating two-soliton bound states and higher-order necklace solitons in nonlocal nonlinear media and demonstrate that they all can be stabilized for sufficiently high degree of nonlocality. Our results indicate that such rotating dipole solitons and higher-order necklace solitons should be observed experimentally in thermal nonlinear media.

2. Theory

We consider the scalar nonlinear Schrödinger equation [1] describing the propagation of paraxial optical beams

$$i\frac{\partial E}{\partial z} + \nabla^2 E + N(I,\vec{r})E = 0, \quad (1)$$

where z and $\vec{r} = (x, y)$ are the propagation and transverse coordinates, respectively. The nonlinearity-induced change of the refractive index n depends on the light intensity $I \equiv |E|^2$ via the following phenomenological nonlocal relation:

$$N(I,\vec{r}) = \int R(|\vec{r}-\vec{\rho}|)I(\vec{\rho})\,\mathrm{d}\vec{\rho}. \quad (2)$$

The actual form of the response function $R(r)$ is determined by the details of the physical process responsible for the medium nonlinearity. Here, we will consider a Gaussian model of nonlocal response

$$R(r) = (\pi\sigma^2)^{-1}\exp(-r^2/\sigma^2), \quad (3)$$

where σ measures the degree of nonlocality.

Azimuthon solutions can be found numerically by two-dimensional relaxation methods [20]. However, this approach represents major difficulties due to a poor convergence for complex envelopes with singular phase profiles. Therefore, below we employ an approximate but much simpler variational technique which enables to obtain an acceptable approximation to the exact solutions [19]. To this end, we introduce the following ansatz:

$$E = \frac{\sqrt{\pi}}{\sigma}U\left(\frac{r}{\sigma}\right)[\cos(m\varphi) + \mathrm{i}p\sin(m\varphi)]\mathrm{e}^{\mathrm{i}(k/\sigma^2)z}, \quad (4)$$

where φ is the angular coordinate and parameter p $(0 \leqslant p \leqslant 1)$ determines the modulation depth (contrast) of azimuthon intensity. After substituting this ansatz into the averaged Lagrangian associated with Eq. (1) [21], we arrive at the equation for the radial envelope $U(r)$,

$$-kU + \frac{\mathrm{d}^2U}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}U}{\mathrm{d}r} - \frac{m^2}{r^2}U + UN(U^2, r) = 0, \quad (5)$$

where the nonlinear term is given by the expression

$$N(U^2, r) = \pi(1+p^2)\mathrm{e}^{-r^2}\int_0^\infty \rho\,\mathrm{d}\rho\,\mathrm{e}^{-\rho^2}U^2(\rho) \times \left\{I_0(2r\rho) + \frac{1}{2}\left(\frac{1-p^2}{1+p^2}\right)^2 I_{2m}(2r\rho)\right\} \quad (6)$$

and I_n is the modified Bessel function of the first kind. In the limit $p \to 1$, the envelope becomes radially symmetric and Eq. (4) describes a vortex soliton. On the other hand, for $p \to 0$, the solution represents the *scalar multipole solitons*, or optical necklaces, known to be radially unstable in local nonlinear media [16].

By applying the numerical shooting technique [17] to Eq. (5) combined with an iteration procedure, we find the profiles of stationary solutions with different values of the parameters p and k. Our results are summarized in Fig. 1. Plots (a,b) show the radial envelopes (solid) and corresponding profiles of the refractive index (dashed) for dipole ($p = 0$) and vortex ($p = 1$) solitons for (a) low and (b) high degree of the nonlocal response. Fig. 1(c) depicts the dependence of the soliton power on the propagation constant k, for different values of p. Fig. 1(d–f) present the spatial distributions of intensity (top) and phase (bottom) distributions for dipole, azimuthon, and vortex solitons, respectively. These plots clearly show the impact of the depth modulation (p) on the intensity structure of the solitons. They also demonstrate that the transition from a dipole soliton to a radially symmetric vortex soliton is achieved by a continuous azimuthal transformation. Hence, the two-peak azimuthons provide a link between soliton spiraling, or the soliton two-body problem, and optical vortices, singular beams with phase dislocations.

In order to study the stability of stationary solutions we employ numerical simulations. We integrate Eq. (1) using the split-step fast Fourier transform beam propagation method. As an initial condition, we use the stationary solutions perturbed initially by noise of 10–20% and propagate them over the large distances of $z{\sim}10^2-10^3$ (or even $\sim 10^4$ in some cases).

Fig. 2 illustrates the typical propagation scenarios. When the nonlocality is too weak (a) the dipole soliton breaks into two mutually repelling filaments. This scenario is reminiscent of the azimuthal instability of vortices in local media. When the degree of nonlocality grows [Fig. 2(b)] a low-contrast azimuthon breaks up initially into two filaments which are subsequently forced by the nonlocality-induced waveguide to collide resulting in a formation of a single fundamental soliton. Finally, the case (c) illustrates

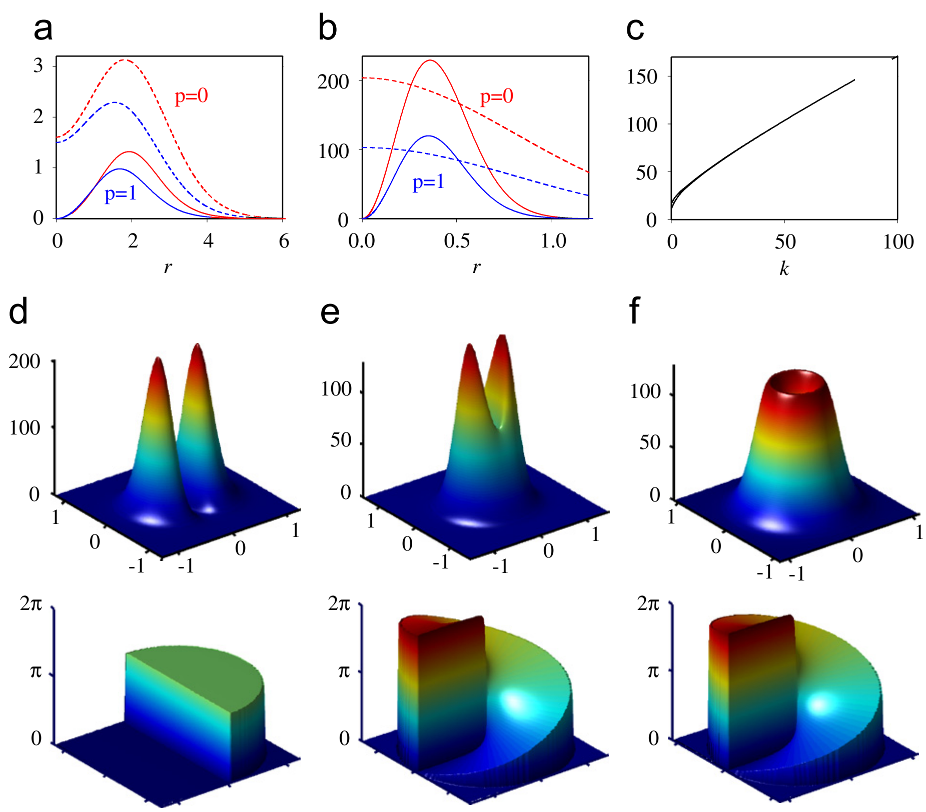

Fig. 1. Dipole solitons in nonlocal Gaussian media: intensity (solid) and corresponding nonlinear response $N(U,r)$ (dashed) for dipole solitons ($p=0$, red) and vortex solitons ($p=1$, blue) for (a) low ($k=1$) and (b) high ($k=70$) degree of nonlocality. In (c) the power of azimuthon vs. propagation constant k is shown. (d–f) Intensity (top) and phase (bottom) for three characteristic cases with $k=70$ and different values of p: (d) $p=0$, (e) $p=1/\sqrt{2}$, and (f) $p=1$.

the stable propagation of a rotating high-contrast azimuthon. Apparently, the effective potential associated with the interplay of nonlocality-mediated attractive and centrifugal repulsive forces attains its minimum enabling the existence of the stable rotating solitons [16]. By testing the stability of various azimuthon solutions in the parameter space (p,k), we determine the stability domain shown in Fig. 2(d). It is clear that the stability threshold decreases for the azimuthons approaching the generic vortex limit ($p=1$). This feature can be associated with the soliton rotation that helps to stabilize the soliton. In addition, while the dipole soliton ($p=0$) experiences mutual repulsion of its out-of-phase lobes, the vortex soliton is radially balanced, and it needs only stabilization against the azimuthal perturbations. Fig. 2(e) depicts the dependence of the angular velocity of azimuthons on the parameter p.

The Gaussian model considered above serves as a phenomenological example of a nonlocal nonlinear medium. While this is a very useful model for the theoretical analysis, it does not describe a specific physical system of a nonlocal optical response. Therefore, below we discuss the properties of the rotating solitons in a thermal medium which represents a real system with nonlocal nonlinearity. In this particular case, the nonlinearity results from a local heating of the medium by the light beam and subsequent change of the refractive index via the thermooptic effect. Therefore, the nonlinear response $N(I)$ of a thermal medium is described by the heat equation which in the steady state has the following form:

$$\frac{\partial^2 N(I)}{\partial x^2} + \frac{\partial^2 N(I)}{\partial y^2} = -\gamma I, \qquad (7)$$

where γ depends on the material parameters such as thermal diffusion constant, linear absorption, and thermo-optic coefficient.

Nonlocality here appears due to a heat flow from a source into the areas of lower temperature. As a result, both the change of temperature and the index of refraction extend far beyond the beam location. Moreover, unlike typical materials where the extent of nonlocality is finite being usually restricted to the immediate surrounding of the light beam, in the thermal media the steady-state temperature and the refractive index distributions are determined not only by the intensity of the beam but also by the boundary conditions. For that reason thermal media are often termed as “infinitely nonlocal”. This aspect of thermal nonlinearity has been exploited in a series of the recent experiments on the soliton interaction and stabilization of vortex and multipole soliton structures in lead glasses [13,22].

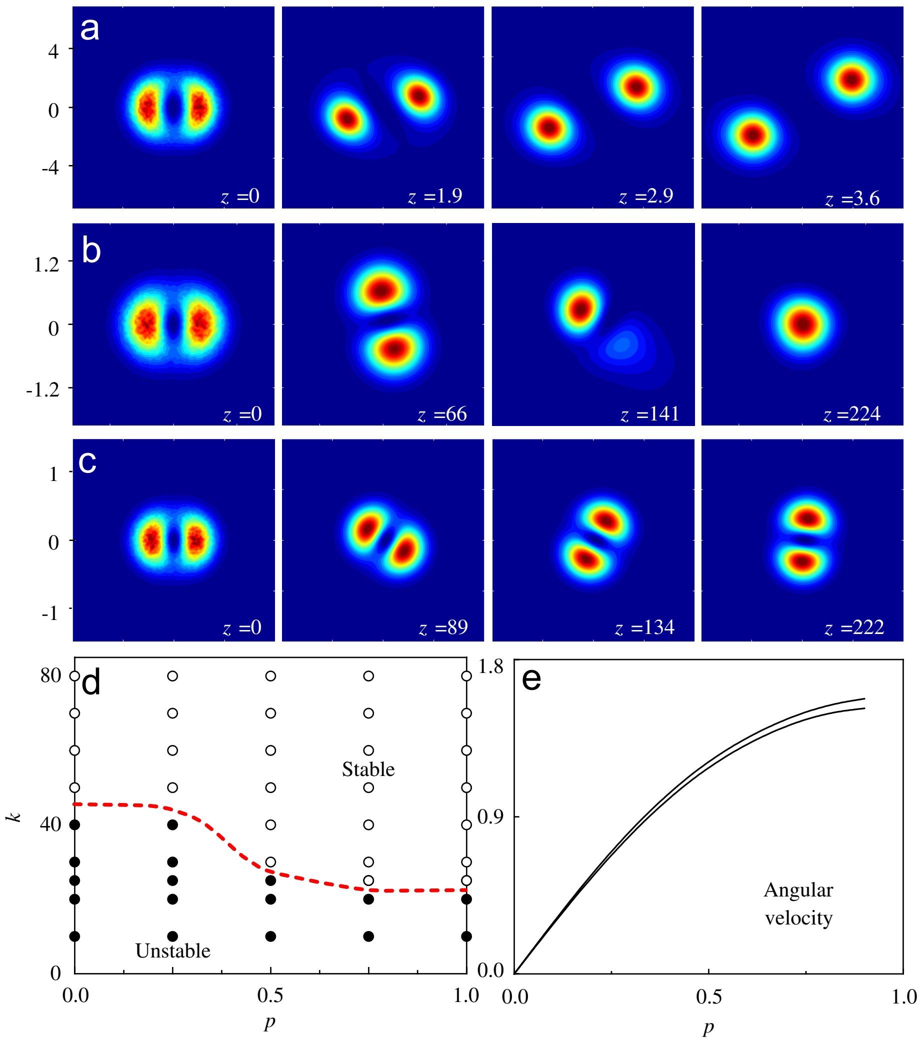

Fig. 2. Unstable and stable dynamics of the dipole solitons in nonlocal media. (a,b) Two different scenarios of instability of rotating dipole solitons with $p = 0.5$ for $k = 1$ and 20, respectively. (c) Stable rotation of the dipole soliton for $p = 0.5$ and $k = 100$. In all cases (a–c) initial noise is 10%. (d) Stability domain of azimuthons on the parameter plane (p, k). (e) Numerically found relation between the angular velocity of dipole solitons and the contrast parameter p. Velocity increases monotonically with the angular momentum S, and it depends only weakly on the power (two curves correspond to $k = 60$ and 80). For the vortex soliton (at $p = 1$) the rotation velocity is an arbitrary free parameter.

Since the solution to the heat equation amounts to solving a boundary-value problem, we resort to numerical simulations in our analysis of the multipole soliton formation. To this end Eq. (1) is solved using the split-step fast Fourier transform technique. At each propagation step the spatial profile of the refractive index is evaluated by solving iteratively the heat equation with the corresponding light intensity distribution as a heat source. We use Eq. (4) to define the initial amplitude of the beam and then propagate the beam over the distance of tens of the diffraction lengths. In physical units, this corresponds to a distance of more than 20 cm for beams of tens of micrometers in diameter. In order to link our simulations to real physical systems, we use the material parameters of lead glasses from Ref. [13]. The thermally induced maximum index change is of the order of 10^{-4} for the beam power in the range of few watts.

In Fig. 3(a), we demonstrate the stable propagation of a rotating dipole soliton. In this particular case, the initial contrast parameter is $p = 0.7$, and the optical power is taken as 5 W. To demonstrate the unique long-range character of thermal nonlocality, in Fig. 3(b) we plot the spatial distribution of the refractive index change. It is clear that while the dipole soliton itself occupies only a finite region, the steady-state refractive index profile extends over the whole computational window.

In order to characterize the stability properties of the rotating dipole solitons, we conduct additional extensive numerical simulations of the soliton propagation varying the initial contrast and input power. The results of these simulations are summarized in Fig. 3(c) which is analogous to that shown above in the case of the Gaussian nonlocal nonlinear model. Each point of this graph represents a rotating dipole soliton with the specific set of

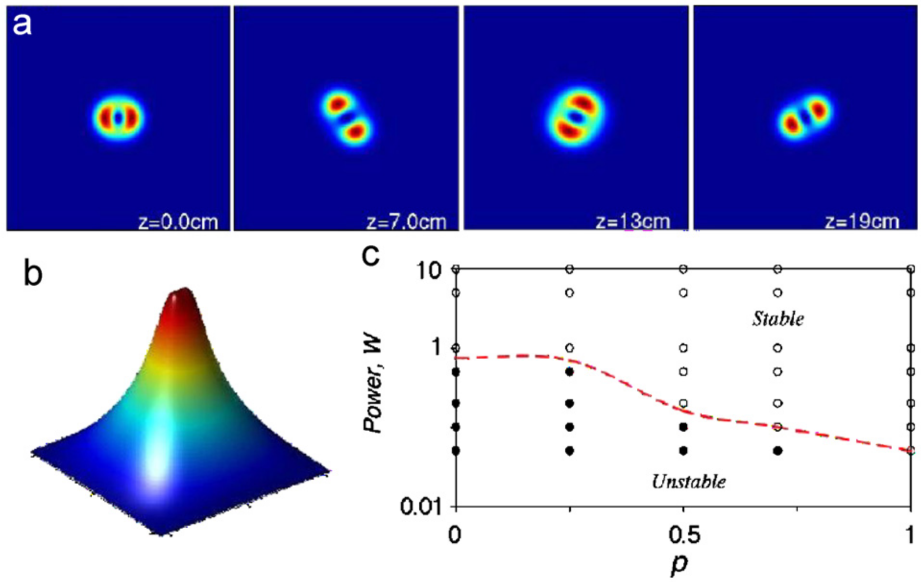

Fig. 3. (a) Example of a stable rotating dipole soliton in a thermal nonlocal nonlinear medium; (b) the spatial profile of the refractive index change induced by a dipole soliton with the contrast parameter $p = 0.7$ and input power 5 W; width of the computational window is $200\,\mu m \times 200\,\mu m$; and (c) stability domain for the rotating dipole solitons for the parameters (p, P).

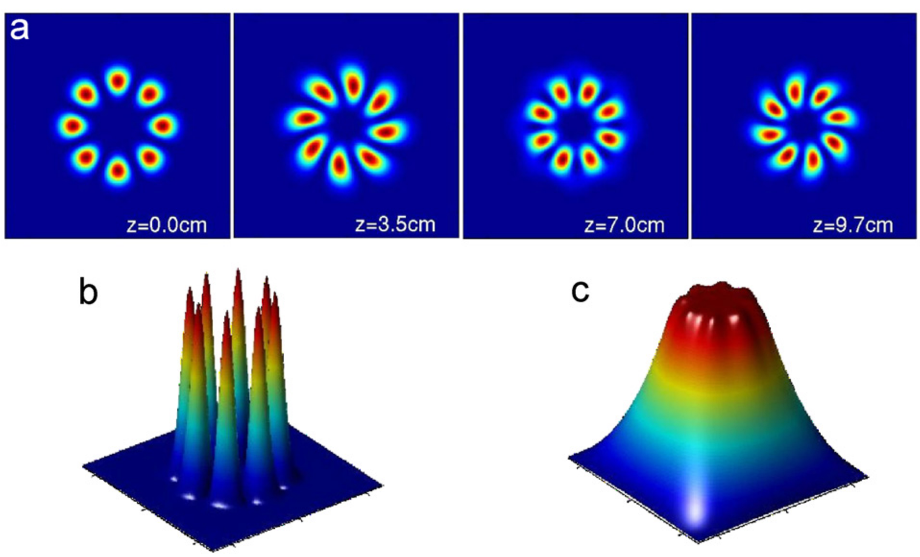

Fig. 4. (a) Example of a stable rotating necklace soliton in a thermal nonlocal medium; input power 5 W; width of the computational window $200\,\mu m \times 200\,\mu m$. (b,c) Surface plots of the intensity and refractive index profile, respectively.

parameters, and its stability is determined numerically by the study of the propagation dynamics. We restrict ourselves to a physically reasonable region of the input powers up to 10 W. We find that in this region, the only relevant instability scenario involves the breakup of the dipole–soliton structure. However, because of a finite size of the computational window and an infinite extent of nonlocality, the complete breakup was never observed. Even if the splinters separate initially, this process is slowed down later by the index gradient in the vicinity of the boundaries. Therefore, the dashed line in Fig. 3(c) represents a qualitative transition between the stable and unstable regimes. Its position was determined from the dependence of separation between the fragments of the dipole soliton as a function of the input power. It should be mentioned that in the recent work [22] the stability of *nonrotating* dipole solitons in thermal media has been investigated, and it was found that all such dipoles are in principle unstable. With higher optical power they experience collapse analogous to the case shown in Fig. 2. Our simulations (not shown) confirm that scenario. However, since the growth rate for this type of instability is very low and collapse becomes important only for very high input powers, we can claim that for typical experimental conditions these azimuthons are *physically* stable.

Stabilizing character of nonlocal nonlinearity suggests that nonlocal media should be able to support higher-order rotating soliton structures. In Fig. 4 we show an example of the stable rotating necklace-type solitons. The initial spin is imposed onto the structure by an initial helical phase modulation. Visible focusing is caused by the fact that the initial intensity pattern does not represent a stationary solution of the nonlinear model.

3. Conclusions

In conclusion, we have demonstrated numerically that a Kerr-type nonlinear optical medium with a Gaussian nonlocal nonlinear response supports the formation of stable rotating dipole solitons (dipole 'azimuthons') for sufficiently high degree of nonlocality. Similar rotating dipole solitons and higher-order localized structures ('necklace solitons') have been also predicted to exist in another, more realistic optical media with thermal nonlinearity. We have demonstrated also that nonlocality provides an effective stabilization mechanism for higher-order localized structures such as soliton necklaces against azimuthal modulational instability.

Acknowledgment

This work was supported by the Australian Research Council.

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