JOSA COMMUNICATIONS

Communications are short papers. Appropriate material for this section includes reports of incidental research results, comments on papers previously published, and short descriptions of theoretical and experimental techniques. Communications are handled much the same as regular papers. Proofs are provided.

Analysis of eigenfields in the axicon-based Bessel–Gauss resonator by the transfer-matrix method: comment

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Numerical calculations recently published [J. Opt. Soc. Am. A **23**, 912 (2006)] on the eigenmodes in axicon-based Bessel–Gauss resonators revealed significant inconsistencies regarding the modal patterns, the mode losses, the mode ordering by loss, and the intracavity field distributions. We show that the results are inaccurate mainly because (a) it was overlooked in the derivation of the matrix equations that light crosses twice through the axicon in a complete round-trip and (b) the numerical method used to evaluate the diffraction integral equations cannot resolve the eigenvalues and eigenfields for the given resonator configuration.

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In a recent paper,[1] the authors claimed the introduction of a transfer-matrix method for finding the eigenmodes of an axicon-based Bessel–Gauss resonator (ABGR). Since the authors adopted a resonator configuration identical to that which we analyzed previously in Ref. 2, we would expect the two papers to yield very similar results. However, after comparing the two papers, we noted that in Ref. 1, the reported modal patterns, the mode losses, the mode ordering by loss, and the intracavity field distributions exhibit several inconsistencies. In this comment, we show that the results presented in Ref. 1 are inaccurate.

In Ref. 1, the authors fail to consider that in a complete round-trip through the ABGR the light crosses twice through the refractive axicon.[2–8] This is a contradictory situation, because this fact was pointed out twice throughout the paper, but it was overlooked in the derivation of the matrix equations. In Eqs. (15) and (19) the phase factor $\exp(-ik\theta_0 na/M)$ represents only a single crossing through the axicon.[2] This error leads to the fact that the cavity now sees an equivalent axicon with wedge angle divided by 2, and eventually to the violation of the condition[2]

$$L = \frac{a}{2(n-1)\alpha}, \qquad (1)$$

which relates the cavity length L to the wedge angle α, the refraction index n, and the aperture radius a of the axicon. Since it is natural to suspect that the missing factor of 2 in Eqs. (15) and (19) is merely a misprint, we computed the mode patterns and mode losses of the ABGR by considering deliberately that light crosses the axicon only once. We employed two independent approaches: the transfer-matrix method explained in Ref. 2 and the Fox–Li method based on a Hankel transform propagator.[9,10] The two methods yielded the same erroneous results reported in Table 1 of Ref. 1. On the other hand, we also implemented the algorithm proposed in Ref. 1 but now considering correctly that light crosses twice through the axicon; in this way we obtained the correct losses and field patterns reported in Table 2 and Figs. 4–6 of Ref. 2. Therefore, the origin of the discrepancies between Refs. 1 and 2 is not an apparent omission in Ref. 2 of the diffraction loss resulting from the finite aperture of the output coupler, as the authors of Ref. 1 mistakenly conjectured in Sec. 4.

The implementation of Eqs. (15) and (19) in Ref. 1 leads to several inconsistencies. For example, in Table 1, it is reported that as the order of the mode increases, the corresponding mode loss decreases, implicating that (even for a flat output mirror) the higher-order mode (2,0) is more dominant than the mode (0,0), which indeed represents the fundamental J_0 Bessel–Gauss beam. This loss behavior is incompatible with results in existing literature and experimental evidence.[2–7,11] Additionally, the transverse amplitude distributions shown in Figs. 3–6 exhibit unexpected radial zero-intensity lines that break the expected azimuthal symmetry imposed by the general form of the fields $E(r)\exp(in\varphi)$ given in Eqs. (7) of Ref. 1.

To evaluate numerically the diffraction integral equations in Ref. 1, the authors applied a simple Riemann sum based on dividing the integration range into $M=256$ points equally spaced. However, it is known that the applicability of the Riemann sum is limited to smooth and well-behaved integrands[12]; consequently, in situations where the field variations become faster, the accuracy of the algorithm proposed in Ref. 1 decreases dramatically. For the particular resonator data used in the paper, we found that 256 points are not enough to resolve the eigenvalues of the first and the second radial eigenmodes. Thus, we verified that the plot in Fig. 7(a) corresponds not to the fundamental mode but to the second radial mode across the axicon. By testing the Riemann sum method, we have found that at least $M=2100$ points are needed to get correct results for the ABGR with $R=10L$, which constitutes a significant difference in computational effort with respect to the 200 points used in Ref. 2 employing the Gauss–Legendre quadrature. The advantages of Gauss–Legendre quadrature over more rudimentary numerical methods of integration for extracting eigenmodes of optical resonators using matrix methods have long been well established.[13–15]

The intracavity field distributions and the output beams shown in Figs. 8 and 9 of Ref. 1 are also inaccurate because the initial fields (at $z=0$ in all plots) cannot propagate in such a way that their overall intensities increase hundreds of times without violating the energy conservation principle. The correct intracavity field distributions and output field propagations of an ABGR are shown in Figs. 8 and 11 of Ref. 2, easily identifiable are not only the strong link between the intracavity distributions and the ray-optics approach but also the conical region where the output beam preserves invariance, the radial Bessel–Gaussian oscillations whose amplitude and frequency agrees with the theoretical model, the expected annular structure of the far-field, and the maximum nondiffractive propagation distance of the output beam.

As a concluding comment, we do not see a reason to claim the introduction of a new transfer-matrix method for extracting the eigenmodes of the ABGR, because the simple product of the two partial matrices [Eqs. (1) and (2) in Ref. 1] yields *exactly* the transfer matrix for the whole cavity that we previously derived and applied in Ref. 2 [Eq. (6)]. Additionally, the discrepancies between Refs. 1 and 2 are not due to the apparent omission of the diffraction losses at the output coupler, as we have shown, but to conceptual and numerical errors in the analysis of the ABGR in Ref. 1; moreover, the use of a Riemann sum, instead of improving, decreases the accuracy of the original Gauss–Legendre method applied in Ref. 2.

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REFERENCES

1. D. Ling, J. Li, and J. Chen, "Analysis of eigenfields in the axicon-based Bessel–Gauss resonator by the transfer-matrix method," J. Opt. Soc. Am. A **23**, 912–918 (2006).
2. J. C. Gutiérrez-Vega, R. Rodríguez-Masegosa, and S. Chávez-Cerda, "Bessel–Gauss resonator with spherical output mirror: geometrical and wave-optics analysis," J. Opt. Soc. Am. A **20**, 2113–2122 (2003).
3. R. I. Hernández-Aranda, S. Chávez-Cerda, and J. C. Gutiérrez-Vega, "Theory of the unstable Bessel resonator," J. Opt. Soc. Am. A **22**, 1909–1917 (2005).
4. M. Alvarez, M. Guizar-Sicairos, R. Rodríguez-Masegosa, and J. C. Gutiérrez-Vega, "Construction and characterization of CO_2 laser with an axicon based Bessel–Gauss resonator," in *Laser Resonators and Beam Control VIII*, A. V. Kudrayashov and A. H. Paxton, eds., Proc. SPIE **5708**, 323–331 (2005).
5. J. Rogel-Salazar, G. H. C. New, and S. Chávez-Cerda, "Bessel–Gauss beam optical resonator," Opt. Commun. **190**, 117–122 (2001).
6. A. N. Khilo, E. G. Katranji, and A. A. Ryzhevich, "Axicon-based Bessel resonator: analytical description and experiment," J. Opt. Soc. Am. A **18**, 1986–1992 (2001).
7. P. Muys and E. Vandamme, "Direct generation of Bessel beams," Appl. Opt. **41**, 6375–6379 (2002).
8. C. L. Tsangaris, G. H. C. New, and J. Rogel-Salazar, "Unstable Bessel beam resonator," Opt. Commun. **223**, 233–238 (2003).
9. A. G. Fox and T. Li, "Resonant modes in a maser interferometer," Bell Syst. Tech. J. **40**, 453–488 (1961).
10. M. Guizar-Sicairos and J. C. Gutiérrez-Vega, "Computation of quasi-discrete Hankel transforms of integer order for propagating optical wave fields," J. Opt. Soc. Am. A **21**, 53–58 (2004).
11. A. Hakola, S. C. Buchter, T. Kajava, H. Elfstrom, J. Simonen, P. Paakkonen, and J. Turunen, "Bessel–Gauss output beam from a diode-pumped Nd:YAG laser," Opt. Commun. **238**, 335–340 (2004).
12. W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes in C: The Art of Scientific Computing* (Cambridge U. Press, 1992).
13. R. L. Sanderson and W. Streifer, "Comparison of laser mode calculations," Appl. Opt. **8**, 131–136 (1969).
14. W. D. Murphy and M. L. Bernabe, "Numerical procedures for solving nonsymmetric eigenvalue problems associated with optical resonators," Appl. Opt. **17**, 2358–2365 (1978).
15. W. P. Latham Jr. and G. C. Dente, "Matrix methods for bare resonator eigenvalue analysis," Appl. Opt. **19**, 1618–1621 (1980).