Comment on "Exact solution of resonant modes in a rectangular resonator"

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We comment on the recent Letter by J. Wu and A. Liu [Opt. Lett. **31**, 1720 (2006)] in which an exact scalar solution to the resonant modes and the resonant frequencies in a two-dimensional rectangular microcavity were presented. The analysis is incorrect because (a) the field solutions were imposed to satisfy simultaneously both Dirichlet and Neumann boundary conditions at the four sides of the rectangle, leading to an overdetermined problem, and (b) the modes in the cavity were expanded using an incorrect series ansatz, leading to an expression for the mode fields that does not satisfy the Helmholtz equation. © 2006 Optical Society of America

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In a recent Letter[1] an exact scalar solution to the resonant modes and the resonant frequencies in a two-dimensional (2D) rectangular microcavity was reported. The solutions and results reported in the Letter are argued to be incorrect.

First, it was claimed, "…In this Letter we report for the first time to our knowledge, an exact solution to the resonant modes and the resonant frequencies in 2D rectangular microcavity." In fact, the solutions of the scalar Helmholtz equation in Cartesian coordinates for 2D and 3D rectangular closed cavities are reported in many textbooks on mathematical methods[2,3] and electromagnetism.[4] Even more recently, the scalar and vector modal analysis of rectangular and square resonators with metallic and dielectric walls have been studied theoretically and experimentally for a variety of applications; see, for example, Refs. 5–11. In these works, the cavity modes are written in closed and natural form using the eigenfunctions of the Helmholtz equation in Cartesian coordinates, instead of the eigenfunctions of the Helmholtz equation in cylindrical coordinates, as pretended by the authors in Eqs. (2) of the Letter.

There are two fundamental mistakes in the paper: First, the scalar mode solutions are restricted to satisfy simultaneously both Dirichlet and Neumann boundary conditions [Eqs. (3) of the Letter] at the four sides of the rectangle. Independent of the fact that there is not a physical reason to assume both conditions simultaneously, from a mathematical point of view it is well known that specifying both Dirichlet and Neumann boundary conditions to the scalar Helmholtz equation in a closed region, leads to an overdetermined problem[2,3] whose only solution is the trivial solution $\varphi(x,y)=0$.

Second, the scalar function in Eq. (6), which is claimed to represent the exact modes inside the cavity, is incorrect because it does not satisfy the scalar Helmholtz equation, as can be easily demonstrated by direct substitution of Eq. (6) into Eq. (1). Even if the boundary conditions had been correct, the crucial error in the Letter is that an eigenmode of a rectangular cavity with sides at $x=\{0,a\}$ and $y=\{0,b\}$ [e.g., $\varphi_{m,n}=\sin(m\pi x/a)\sin(n\pi y/b)$] cannot be expanded using a series of the form $\varphi=\sum_{l=-\infty}^{\infty}A_l J_l(kr)\cos(l\theta)$. Excepting the trivial case $\varphi=0$, there is not way that this series can vanish at the four sides of the rectangle for a single value of k. The fact that the authors re-express the series (2a) in Cartesian coordinates and further expand it using the addition formula in Eq. (4) does not change this conclusion. Although not practical for a rectangular symmetry, a correct expansion for the rectangular mode $\varphi_{m,n}$ using cylindrical eigenmodes should have involved terms of the general form $J_l(kr)\exp(il\theta)$. By specifying Dirichlet boundary conditions and applying the symmetries present in the problem, we can arrive after some algebraic manipulations at the correct series:

$$\varphi_{m,n}=\sum_{l=1}^{\infty}[2(-1)^{l+1}\sin(2l\beta)]J_{2l}(k_{m,n}r)\sin(2l\theta), \quad (1)$$

where $\beta=\arctan(na/mb)$ and $k_{m,n}=\pi\sqrt{(m/a)^2+(n/b)^2}$.

All the following results derived in the Letter come from the interpretation and plotting of Eq. (6) and consequently lead to inconsistencies. For example, Eqs. (8) are used to determine the eigenfrequencies for the rectangular cavity; however, if in principle the length sides a and b of the rectangle are arbitrary, then Eqs. (8) cannot be satisfied simultaneously for the same value of k. The only way to satisfy Eqs. (8) would be assuming a square cavity $a=b$ and mode indices (m,n) such $n-m=m$. Note that the examples presented in the Letter correspond indeed to a square cavity with $(m,n)=(0,0)$ and $(m,n)=(4,2)$.

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