Exact Unified Theory of Scalar Paraxial and Nonparaxial Beams

G. Rodríguez-Morales and S. Chávez-Cerda

Recent technological developments have made possible the construction of miniature structures of the order of several nanometers. Examples are microdisk lasers and nanolasers and the so-called "lab on a chip," which is a microchip with a built-in nanosized light source and sensors to perform instant and detailed analyses for chemistry, biology, and medical studies. At these dimensions, the transverse size of laser light beams can be of the order of a few wavelengths, and their main features might behave differently from what we are used to with paraxial beams. For example, electromagnetic vector effects start to become an important issue that could manifest itself in different ways.

There are two methods commonly used to describe nonparaxial beams, or beams which have a diameter of a few wavelengths. One uses the addition of correction terms to paraxial beam solutions and the other, the virtual point source method, assumes a point source placed in a complex space.[1,2] A drawback of the virtual source solutions is that they carry an inherent singularity that makes them inadequate to describe propagating fields near the origin or focal source point. To eliminate this problem, nonsingular spherical stationary waves have been used, but even these nonsingular solutions are problematic because infinite energy is required to realize them physically.

In a recent publication, we investigated the traveling-wave properties of the Helmholtz equation in spheroidal coordinates and found a precise unified formulation for beam propagation that is physically consistent for paraxial and nonparaxial regimes.[3]

By determining that the beam-width evolution of paraxial Gaussian modes follows a hyperbola, we found the relation between the physical parameters of Gaussian modes and those of nonparaxial spheroidal beam-like solutions. Then, by using the accepted criteria for the maximum angle that a paraxial beam can diffract, we defined a threshold between nonparaxial and paraxial beams.[4] As expected, within the paraxial limit the oblate spheroidal solutions investigated tend to the known Laguerre–Gaussian beams. By extension, a similar analysis of the two-dimensional Helmholtz equation yields paraxial Hermite–Gaussian modes from nonparaxial Mathieu modes.

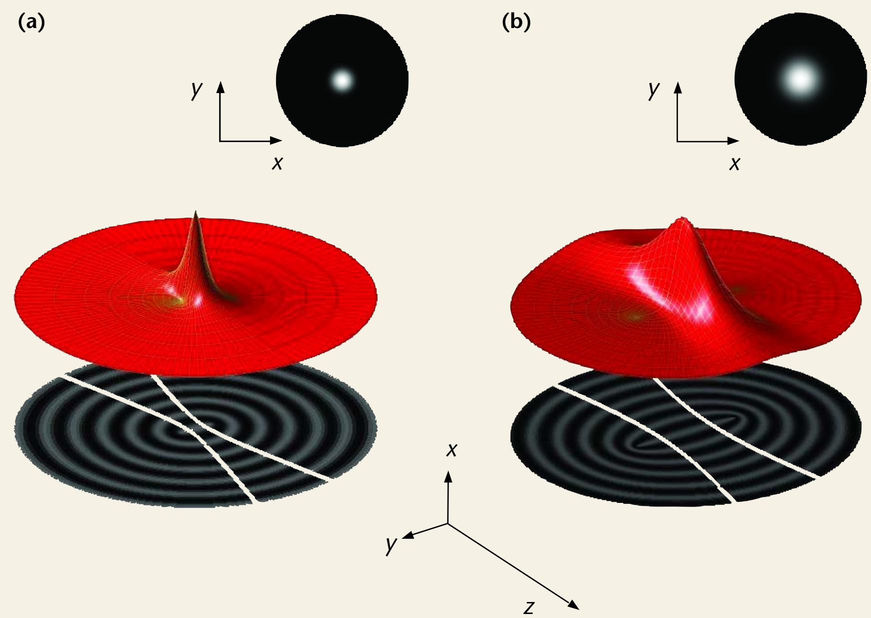

Figure 1. Zero-order spheroidal mode: (a) nonparaxial and (b) paraxial. The red surface plot corresponds to the intensity distribution, the bottom plot to the phase fronts. The beam width hyperbolic behavior is also shown. Each inset depicts the transverse profile at the beam waist.

Figure 1 shows the zero-order spheroidal scalar mode for (a) nonparaxial and (b) paraxial situations. The top surface plot represents the intensity distribution and the bottom density plot shows the structure of the wave fronts. The inset shows the transverse intensity at plane $z = 0$. In the paraxial case, when the intensity distribution is compared with the Laguerre–Gaussian beam solutions, the differences are negligible.

At this stage we have presented only the scalar solutions, however, it is well known that the vector solutions of the Helmholtz equation can be easily found after the scalar solutions are obtained. We expect that the traveling-wave beam-like solutions of the Helmholtz equation presented here shed new light on the physics of beam propagation of a few wavelengths. For example, paraxial and nonparaxial spheroidal beams are also structurally stable, i.e. shape invariant, in the same manner as the paraxial Laguerre–Gaussian beam.

G. Rodríguez-Morales (gustavorm@itesm.mx) is with the Photonics and Mathematical Optics Group, Tecnológico de Monterrey, Monterrey, México. S. Chávez-Cerda (sabino@inaoep.mx) is with the Instituto Nacional de Astrofísica Óptica y Electrónica, Puebla, México.

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