Observation of Ince–Gaussian modes in stable resonators

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Received April 5, 2004

We report what is to our knowledge the first observation of Ince–Gaussian modes directly generated in a stable resonator. By slightly breaking the symmetry of the cavity of a diode-pumped Nd:YVO$_4$ laser and its pump beam configuration we were able to generate single high-order Ince–Gaussian modes of high quality. The observed transverse modes have an inherent elliptic structure and exhibit remarkable agreement with theoretical predictions.

OCIS codes: 260.1960, 350.5500, 140.3410, 140.3530, 140.3300.

The transverse field structure of modes supported by a stable resonator has been investigated with analytical, numerical, and experimental techniques. Besides the well-known Hermite–Gaussian modes (HGMs) and Laguerre–Gaussian modes (LGMs),[1] in recent papers the existence of Ince–Gaussian modes (IGMs), which constitute the third complete family of transverse eigenmodes of stable resonators, was theoretically demonstrated.[2,3] These new modes are exact and orthogonal solutions of the paraxial wave equation in elliptic coordinates and may be considered continuous transition modes between HGMs and LGMs.

In this Letter we report what we believe is the first observation of IGMs directly generated in a simple stable resonator. The experiment employs a Nd:YVO$_4$ crystal as an active medium pumped by a laser diode at 808 nm. We found that when the symmetry of the resonator is slightly broken the cavity is capable of emitting elliptical transverse modes that can be identified as high-order IGMs of high quality. The experimental results exhibit good agreement with the theoretical predictions and reveal that the astigmatism of the resonator leads to the IGMs that are azimuthal stationary waves in contrast to traveling rotating waves.

The propagating and resonating characteristics of the IGMs have been discussed in detail in Refs. 2 and 3; it is the notation of those references for the IGMs that we use. In free space we construct a scalar wave field that satisfies the paraxial wave equation and propagates along the positive z axis of an elliptic coordinate system $\mathbf{r} = (\xi, \eta, z)$. We define the elliptic coordinates in a transverse z plane as $x = f(z)\cosh\xi\cos\eta$, $y = f(z)\sinh\xi\sin\eta$, and $z = z$, where $\xi \in [0, \infty)$ and $\eta \in [0, 2\pi)$ are the radial and angular elliptic variables, respectively. Semifocal separation $f(z)$ of the system diverges in the same way as the width of a Gaussian beam, i.e., $f(z) = f_0 w(z)/w_0$, where f_0 and w_0 are the semifocal separation and the beam width at the $z = 0$ plane, respectively, $w(z) = w_0(1 + z^2/z_R^2)^{1/2}$ describes the beam width, $z_R = kw_0^2/2$ is the Rayleigh range, and k is the wave number.

The IGMs with mode numbers p and m and ellipticity parameter ϵ are given by

$$\mathrm{IG}^{e}_{p,m}(\mathbf{r}, \epsilon) = \frac{C w_0}{w(z)} C_p^m(i\xi, \epsilon) C_p^m(\eta, \epsilon) \exp\left[\frac{-r^2}{w^2(z)}\right] \times \exp\left[kz + \frac{kr^2}{2R(z)} - (p+1)\psi_z(z)\right], \quad (1)$$

where C is a normalization constant, the subscript e refers to even modes, and $C_p^m(\cdot, \epsilon)$ are the even Ince polynomials[3,4] of order p, degree m, and parameter ϵ. We obtain odd IGMs, $IG^{o}_{p,m}(\mathbf{r}, \epsilon)$, by writing the odd Ince polynomials $S_p^m(\cdot, \epsilon)$ and the odd normalization constant S instead of the even ones. In Eq. (1), r is the radial distance from the z axis, $R(z) = z + z_R^2/z$ is the radius of curvature of the phase front, and $\psi_z(z) = \arctan(z/z_R)$.

The physically important parameters for describing the transverse structure of the IGMs are ellipticity ϵ, waist spot w_0, and the semifocal separation f_0. These parameters are not independent, but they are related by $\epsilon = 2f_0^2/w_0^2$. Whereas dimensionless parameter ϵ adjusts the ellipticity of the mode, the parameters w_0 and f_0 scale its physical size. When $\epsilon \to 0$, the IGMs tend to LGMs, and, when $\epsilon \to \infty$, the IGMs tend to HGMs. Several theoretical transverse shapes of even and odd low-order IGMs at the waist plane $z = 0$ are shown in Ref. 3.

For the experiment we use a self-built diode-pumped solid-state laser; see Fig. 1. This laser was originally designed to generate high-order LGMs with high fidelity. The active medium is Nd:YVO$_4$ crystal, pumped at 808 nm by a laser diode. The Nd:YVO$_4$

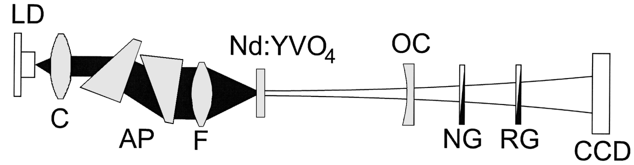

Fig. 1. Experimental arrangement for the observation of IGMs in a laser cavity: LD, $\lambda = 808$ nm pump laser diode; C, collimating lens; AP, anamorphic prism pair; F, focusing lens; Nd:YVO$_4$, laser crystal; OC, 97% output coupler; NG, neutral glass filter; RG, colored glass filter; CCD, camera.

0146-9592/04/161870-03$15.00/0

crystal with 2% doping is 1 mm long, with a high-reflection coating at emission wavelength $\lambda_0 = 1064$ nm and an antireflection coating for the pump wavelength on one facet and an antireflection coating for both wavelengths on the second facet. We use an anamorphic prism pair to reduce the astigmatism of the pump beam, which is then focused by a lens with a 50-mm focal length to a tight focus of approximately 80-μm diameter inside the Nd:YVO_4 crystal. Pump power is 100–300 mW, and output power is of the order of $P_{\mathrm{out}} = 20$ mW for low-order modes.

The clear aperture of the copper heat sink holding the Nd:YVO_4 crystal has a diameter of 1.6 mm, which is large compared with the diameter $w_0 \approx 200$ μm of the fundamental Gaussian mode at the crystal. We use two different output couplers with radii of curvature $R = 75$ mm and $R = 300$ mm and reflectivity of 97% at the wavelength λ_0. The clear diameter of the output couplers is 10 mm, again large compared with the fundamental beam diameter $w \approx 600$ μm at this position. After adjusting for azimuthal symmetry, one could switch the laser from fundamental mode operation to higher-order LGMs simply by increasing the pump power. The mechanism here is that the diameter of the central spot of the higher-order modes is smaller than the fundamental mode and thus is more efficiently pumped by the tight pump focus.[5] We were able to achieve stable lasing for the five lowest-order LGMs.

To generate IGMs we slightly broke the symmetry of the resonator by shifting the output coupler sideways by several tens of micrometers. Then the center of curvature was shifted with respect to the axis given by the pump beam spot and the surface normal of the Nd:YVO_4 crystal that formed the high-reflectivity mirror. Larger astigmatism, for example, introduced by tilting of the output coupler, forced the laser to HGM operation. For a spherical mirror tilting and shifting are in principle identical, but tilting shifts the center of curvature much faster out of the central axis of the resonator.

Figure 2(a) shows the measured intensity distributions of several even high-order IGMs for different pairs p and m and ellipticities ϵ. The corresponding

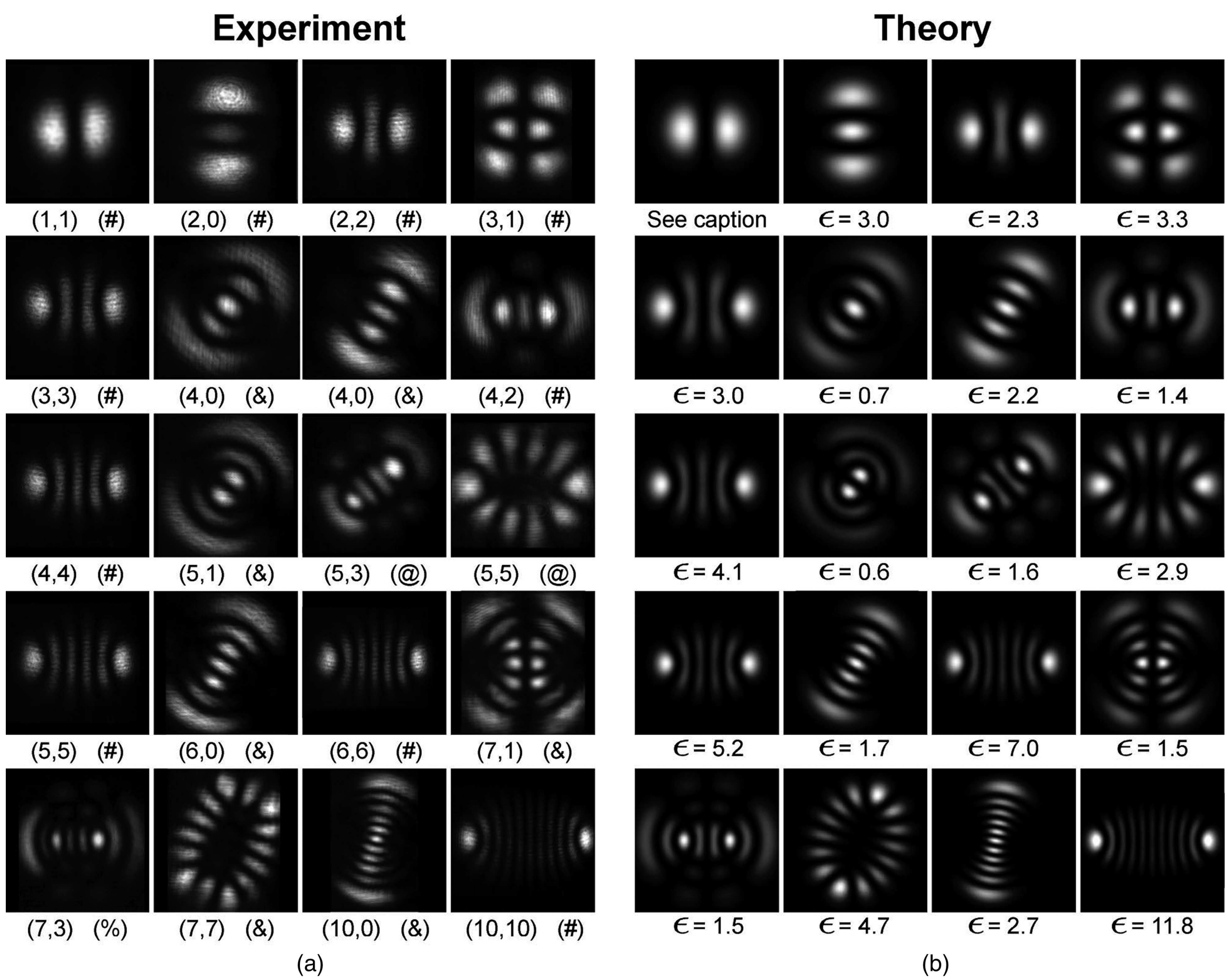

Fig. 2. (a) Beam intensity patterns of even IGMs, $\mathrm{IG}^{e}_{p,m}(\mathbf{r}, \epsilon)$, measured with the CCD camera and (b) their theoretical predictions. Mode numbers p and m are shown in (a), and ellipticity ϵ in (b). The shape of $\mathrm{IG}^{e}_{1,1}(\mathbf{r}, \epsilon)$ is independent of ϵ. When needed, some theoretical patterns were rotated to facilitate comparison with the measured patterns. The experimental setups are as follows: (#) $R_o = 30$ cm, $L = 29.5$ cm, without a cross hair; (&) equal but with a cross hair; (%) $R_o = 30$ cm, $L = 10$ cm, without a cross hair; (@) $R_o = 7.5$ cm, $L = 6.8$ cm, without a cross hair.

theoretical predictions are shown in Fig. 2(b). Radius of curvature R_o of the output coupler and resonator length L employed for obtaining each mode are included in the figure caption. When indicated, an additional cross hair was introduced into the cavity to force the laser to a different order mode. The experimental patterns were measured with a charge-coupled device (CCD) at a distance of 5 cm behind the output coupler, with only a neutral glass filter to adapt the beam power to the CCD sensitivity and a Schott RG1000 filter to suppress the remaining pump light. In general the patterns exhibit a tendency to be aligned to the x or y axis when the output coupler is shifted in this way.

The theoretical patterns shown in Fig. 2(b) were plotted from Eq. (1) by choice of parameters ϵ and w for the best fit with the experimental results. In all cases m corresponds to the number of hyperbolic nodal lines, whereas $(p - m)/2$ is the number of elliptic nodal lines. Beams with higher indices have typically larger extents than those with lower indices. The experimental results reveal that the astigmatism introduced into the resonator causes the IGMs to be azimuthally stationary waves instead of traveling rotating waves.[2,3] We emphasize the excellent agreement between the measured and theoretical patterns shown in Fig. 2.

The ellipticity and mode numbers of the lasing beams depend both on the size of the pump beam focus relative to the beam waist and on the astigmatism of the resonator. We performed measurements at different z planes to corroborate that the transverse shape is affected by scaling factor $w_0/w(z)$ but otherwise maintains its profile. Regardless of the indices and the spatial rotation of the patterns, the polarization of the output beam is always linear and parallel to the polarization of the pump beam.

The Guoy phase shift of the IGMs is given by $(p + 1)\arctan(z/z_R)$; consequently the phase velocity increases with increasing order number p. In resonators this effect leads to differences in the resonance frequencies of the various IGMs of oscillation. For a plano–concave mirror cavity of length L the transverse mode frequency spacing between consecutive IGMs is given by

$$\Delta\nu = (c/2\pi L)\cos^{-1}[(1 - L/R_o)^{1/2}], \quad (2)$$

where c is the velocity of light. Without adjustment of the resonance conditions, the cavity will generate complicated patterns as a result of the incoherent superposition of fundamental modes. Because IGMs, LGMs, and HGMs form three complete and equally valid families for expanding an arbitrary paraxial field, the generation of one family of modes in the resonator could be interpreted as a coherent sum of degenerate modes of any other family simultaneously excited in the cavity. For example, the formation of the $\mathrm{IG}^e_{4,4}(\mathbf{r}, \epsilon = 4.1)$ mode shown in Fig. 2 in terms of LGMs $\mathrm{LG}^e_{n,l}(\mathbf{r})$ is given by $0.67\mathrm{LG}^e_{0,4}(\mathbf{r}) - 0.62\mathrm{LG}^e_{1,2}(\mathbf{r}) + 0.41\mathrm{LG}^e_{2,0}(\mathbf{r})$, where we note that all constituent modes have the same number $p = 2n + l = 4$, and the superscript e refers to even LGMs with cosine azimuthal variation.[3] This implies that the modes are still linear and that their properties do not depend on the nonlinearity of the gain medium. We emphasize here that the patterns shown in Fig. 2(a) constitute what we believe is the first experimental evidence that stable resonators can be forced to single IGM operation.

In conclusion, we have reported experimental generation of IGMs in stable laser resonators. The experimental results exhibit remarkable agreement with theory and demonstrate the existence of IGMs, which constitute the third complete family of transverse eigenmodes of stable resonators. We produced the modes in a diode-pumped Nd:YVO_4 laser by slightly breaking the symmetry of the cavity–pump beam configuration. The ellipticity and mode numbers of the lasing beams depended both on the size of the pump beam focus relative to the beam waist and on the astigmatism of the resonator. In this Letter we have reported measurements of even IGMs; however, generation of odd modes should be possible by adjustment of the pump beam shape. Our results extend the fundamental theory of high-order modes in stable resonators by adding the new IGMs to the well-known HGMs and LGMs.

The visit of J. C. Gutiérrez-Vega to the University of Regensburg was made possible by Universitätsstiftung Hans Vielberth. U. T. Schwarz thanks T. Hafner and G. Tremmel for building the diode-pumped solid-state laser. We are grateful for support by Tecnológico de Monterrey Research Chair in Optics grant CAT-007. J. C. Gutiérrez-Vega's e-mail address is juliocesar@itesm.mx.

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