Observation of Ince–Gaussian modes in stable resonators

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We report what is to our knowledge the first observation of Ince–Gaussian modes directly generated in a stable resonator. By slightly breaking the symmetry of the cavity of a diode-pumped Nd:YVO4 laser and its pump beam configuration we were able to generate single high-order Ince–Gaussian modes of high quality. The observed transverse modes have an inherent elliptic structure and exhibit remarkable agreement with theoretical predictions. © 2004 Optical Society of America

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The transverse field structure of modes supported by a stable resonator has been investigated with analytical, numerical, and experimental techniques. Besides the well-known Hermite–Gaussian modes (HGMs) and Laguerre–Gaussian modes (LGMs),1 in recent papers the existence of Ince–Gaussian modes (IGMs), which constitute the third complete family of transverse eigenmodes of stable resonators, was theoretically demonstrated.2,3 These new modes are exact and orthogonal solutions of the paraxial wave equation in elliptic coordinates and may be considered continuous transition modes between HGMs and LGMs.

In this Letter we report what we believe is the first observation of IGMs directly generated in a simple stable resonator. The experiment employs a Nd:YVO4 crystal as an active medium pumped by a laser diode at 808 nm. We found that when the symmetry of the resonator is slightly broken the cavity is capable of emitting elliptical transverse modes that can be identified as high-order IGMs of high quality. The experimental results exhibit good agreement with the theoretical predictions and reveal that the astigmatism of the resonator leads to the IGMs that are azimuthal stationary waves in contrast to traveling rotating waves.

The propagating and resonating characteristics of the IGMs have been discussed in detail in Refs. 2 and 3; it is the notation of those references for the IGMs that we use. In free space we construct a scalar wave field that satisfies the paraxial wave equation and propagates along the positive z axis of an elliptic coordinate system r = (ξ, η, z). We define the elliptic coordinates in a transverse z plane as x = f(ξ)cosh ξ cos η, y = f(ξ)sinh ξ sin η, and z = z, where ξ ∈ [0, ∞) and η ∈ [0, 2π) are the radial and angular elliptic variables, respectively. Semifocal separation f(ξ) of the system diverges in the same way as the width of a Gaussian beam, i.e., f(z) = f0w(z)/w0, where f0 and w0 are the semifocal separation and the beam width at the z = 0 plane, respectively, w(z) = w0(1 + z2/F2)1/2 describes the beam width, zR = kw2/2 is the Rayleigh range, and k is the wave number.

The IGMs with mode numbers p and m and ellipticity parameter ε are given by

\[
\text{IG}_{p,m}^e(r, ε) = \frac{C w_0}{w(z)} \frac{C_p^m(iξ, ε)C_p^m(η, ε)\exp\left[ -\frac{r^2}{w^2(z)} \right]}{\exp\left[ kz + \frac{kr^2}{2R(z)} - (p + 1)\psi_z(z) \right]},
\]

where C is a normalization constant, the subscript e refers to even modes, and \( C_p^m(\cdot, ε) \) are the even Ince polynomials2,4 of order p, degree m, and parameter ε. We obtain odd IGMs, \( \text{IG}_{p,m}^o(r, ε) \), by writing the odd Ince polynomials \( S_p^m(\cdot, ε) \) and the odd normalization constant S instead of the even ones. In Eq. (1), r is the radial distance from the z axis, R(z) = z + zR/z is the radius of curvature of the phase front, and \( \psi_z(z) = \arctan(z/z_R) \).

The physically important parameters for describing the transverse structure of the IGMs are ellipticity ε, waist spot w0, and the semifocal separation f0. These parameters are not independent, but they are related by \( ε = 2f_0^2/w_0^2 \). Whereas dimensionless parameter ε adjusts the ellipticity of the mode, the parameters w0 and f0 scale its physical size. When \( ε → 0 \), the IGMs tend to LGMs, and, when \( ε → ∞ \), the IGMs tend to HGMs. Several theoretical transverse shapes of even and odd low-order IGMs at the waist plane z = 0 are shown in Ref. 3.

For the experiment we use a self-built diode-pumped solid-state laser; see Fig. 1. This laser was originally designed to generate high-order LGMs with high fidelity. The active medium is Nd:YVO4 crystal, pumped at 808 nm by a laser diode. The Nd:YVO4 laser and its laser crystal; OC, 97% output coupler; NG, neutral glass filter; RG, colored glass filter; CCD, camera.

Fig. 1. Experimental arrangement for the observation of IGMs in a laser cavity: LD, λ = 808 nm pump laser diode; C, collimating lens; AP, anamorphic prism pair; F, focusing lens; Nd:YVO4 laser crystal; OC, 97% output coupler; NG, neutral glass filter; RG, colored glass filter; CCD, camera.
crystal with 2% doping is 1 mm long, with a high-
reflection coating at emission wavelength $\lambda_0 = 1064$ nm and an antireflection coating for the pump wavelength on one facet and an antireflection coating for both wavelengths on the second facet. We use an anamorphic prism pair to reduce the astigmatism of the pump beam, which is then focused by a lens with a 50-mm focal length to a tight focus of approximately 80-µm diameter inside the Nd:YVO$_4$ crystal. Pump power is 100–300 mW, and output power is of the order of $P_{\text{out}} = 20$ mW for low-order modes.

The clear aperture of the copper heat sink holding the Nd:YVO$_4$ crystal has a diameter of 1.6 mm, which is large compared with the diameter $w_0 \approx 200$ µm of the fundamental Gaussian mode at the crystal. We use two different output couplers with radii of curvature $R = 75$ mm and $R = 300$ mm and reflectivity of 97% at the wavelength $\lambda_0$. The clear diameter of the output couplers is 10 mm, again large compared with the fundamental beam diameter $w \approx 600$ µm at this position. After adjusting for azimuthal symmetry, one could switch the laser from fundamental mode operation to higher-order LGMs simply by increasing the pump power. The mechanism here is that the diameter of the central spot of the higher-order modes is smaller than the fundamental mode and thus is more efficiently pumped by the tight pump focus. We were able to achieve stable lasing for the five lowest-order LGMs.

To generate IGMs we slightly broke the symmetry of the resonator by shifting the output coupler sideways by several tens of micrometers. Then the center of curvature was shifted with respect to the axis given by the pump beam spot and the surface normal of the Nd:YVO$_4$ crystal that formed the high-reflectivity mirror. Larger astigmatism, for example, introduced by tilting of the output coupler, forced the laser to HGM operation. For a spherical mirror tilting and shifting are in principle identical, but tilting shifts the center of curvature much faster out of the central axis of the resonator.

Figure 2(a) shows the measured intensity distributions of several even high-order IGMs for different pairs $p$ and $m$ and ellipticities $\epsilon$. The corresponding
Theoretical predictions are shown in Fig. 2(b). Radius of curvature $R_o$ of the output coupler and resonator length $L$ employed for obtaining each mode are included in the figure caption. When indicated, an additional cross hair was introduced into the cavity to force the laser to a different order mode. The experimental patterns were measured with a charge-coupled device (CCD) at a distance of 5 cm behind the output coupler, with only a neutral glass filter to adapt the beam power to the CCD sensitivity and a Schott RG1000 filter to suppress the remaining pump light. The theoretical patterns shown in Fig. 2(b) were plotted from Eq. (1) by choice of parameters $\epsilon$ and $w$ for the best fit with the experimental results. In all cases $m$ corresponds to the number of hyperbolic nodal lines, whereas $(p - m)/2$ is the number of elliptic nodal lines. Beams with higher indices have typically larger extents than those with lower indices. The experimental results reveal that the astigmatism introduced into the resonator causes the IGMs to be azimuthally stationary waves instead of traveling rotating waves.\(^3\) We emphasize here that the patterns shown in Fig. 2(a) constitute what we believe is the first experimental evidence that stable resonators can be forced to single IGM operation.

In conclusion, we have reported experimental generation of IGMs in stable laser resonators. The experimental results exhibit remarkable agreement with theory and demonstrate the existence of IGMs, which constitute the third complete family of transverse eigenmodes of stable resonators. We produced the modes in a diode-pumped Nd:YVO\(_4\) laser by slightly breaking the symmetry of the cavity–pump beam configuration. The ellipticity and mode numbers of the lasing beams depended both on the size of the pump beam focus relative to the beam waist and on the astigmatism of the resonator. In this Letter we have reported measurements of even IGMs; however, generation of odd modes should be possible by adjustment of the pump beam shape. Our results extend the fundamental theory of high-order modes in stable resonators by adding the new IGMs to the well-known HGMs and LGMs.

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